Collaboration Report

Department of Physical Sciences

Supernovae Neutrino Oscillations

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Arsh Dhawan

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Abstract

Neutrinos are peculiar, weak interacting particles that lead us further from QFT and Standard Model Physics with their advent of "Beyond Standard Model Physics", on which many leading researchers are working. Neutrinos are observed physically as only left-handed particles, which opens up a huge debate on it's antiparticle. They are also observed to exist in 3 flavours, namely, electron neutrino, muon neutrino, and tau neutrino. Moreover, it has been observed that a neutrino emitted as one flavour can be measured as another, given enough time or distance traveled. This phenomenon has been named as **Neutrino Oscillations**.
Other than that, cosmologists' current proposal for **Core Collapse Models** for stars with mass greater than 8-9 Solar Masses, have a huge dependence on these small particles. In fact, the impact is such that these are the particles which lead to Supernova explosion with explusion of matter, light and neutrinos of order 10^{53}, which inform us beforehand of a SNe, in a small time of 10sec.
During this project, I worked on these topics on length and tried to understand them better.

Introduction

First proposed by Pauli, to explain beta decay in fluorescent materials, neutrinos are small, neutral particles, which interact with matter only via Weak Interacting Forces and thus, are hard to observe.
Experimentally through elastic collision of electron and neutrino, we were able to pin down its Helicity to be left-handed. This, sadly(or happily) lead us further from the Standard Model, as, as of now, no anti-particle of neutrino has been observed.
The presence of neutrino mass further widens the divide and thus, we consider both Majorana and Dirac fermions in our calculations.

The different flavours of neutrinos are observed to interchange into each other, a phenomenon well explained by the theoretical physicists by lepton mixing, where we consider mass states and flavour states, not overlapping but having a mixing angle(θ) between them.

The details of Neutrino oscillations are presented in Chapter 1 of the report.

Chapter 1

Neutrino Oscillations

1.1 Mathematical Liberties and Tools

- $c = 1$
- $E_i = p_i$
- $i\frac{d}{dL}\left|\nu_f\right\rangle = H_f\left|\nu_f\right\rangle$

1.2 Two Flavour Oscillations

[4]

$$|\nu_i[t,\vec{x}]\rangle = e^{-ip_ix}\left|\nu_i[0,\vec{0}]\right\rangle$$

where i=1,2,3, represent the respective mass states.

$U_{\alpha i}$, also called PMNS matrix or MNSP matrix is used to represent lepton mixing between mass state i, and flavour state α.

$$|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle$$

For two flavour,

$$U = \begin{vmatrix} Cos\theta & Sin\theta \\ Sin\theta & Cos\theta \end{vmatrix}$$

Hence,

$$|\nu_e\rangle = Cos\theta|\nu_1\rangle + Sin\theta|\nu_2\rangle$$

which evolves with time by plane wave approximation;

$$|\nu_e\rangle = Cos\theta e^{-ip_1x}|\nu_1\rangle + Sin\theta e^{-ip_2x}|\nu_2\rangle$$

From the second mathematical tool, we can estimate,

$$p_ix = E_it - p_i.x = (E_i - p_{z,o})L$$

$$E_i - p_{z,i} = \frac{(E_i^2 - |\vec{p}|^2)}{E_i + p_{z,i}} = \frac{m_i^2}{2E}$$

Hence,

$$|\nu[L]\rangle = Cos\theta e^{-im_1^2/2E}|\nu_1\rangle + Sin\theta e^{-im_1^2/2E}|\nu_2\rangle$$

$$P_{ee} = |\langle\nu_e|\nu[L]\rangle|^2 = 1 - 2Sin^2 2\theta Sin^2\frac{\Delta m^2 L}{4E}$$

1.3 Three Flavour Oscillations

[1] Due to more masses and possible mixing angles, we can say the probability will depend upon,

- **mass square differences;** $\Delta m^2_{2,1}, \Delta m^2_{3,1}, \Delta m^2_{3,2}$
- **Mixing angles**; $\theta_{12}, \theta_{23}, \theta_{13}$
- CP violating phase δ_{CP}
- E,L,V

We can write the mixing matrix U as,

$$U = O_{23} U_\delta O_{13} U_\delta^\dagger O_{12}$$

where O's are composed of mixing matrices for respective mass states, whereas U_δ account for CP voilation phase and $U_\delta = diag(1, 1, e^{i\delta_{CP}})$.
The orderng is adjusted such as to first account for Solar Neutrinos(1,2), then intermixing of Solar and Atmospheric Neutrinos(1,3), and lastly considering Atmospheric oscillations(2,3). We have arbitarily chosen CP- voilation phase to aid us better in our endeavors.

Using this, one can estimate probability, considering $\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$ and $Sin\theta_{13}$«1, i.e. as pertubations.
Other than this, using Schrodinger equation, we can get respective probabilitues.

$$P_{ee}^{vac} = 1 - \alpha^2 Sin^2 2\theta_{12}\Delta^2 - 4Sin^2\theta_{13}Sin^2\Delta$$

where,

$$\Delta = \frac{\Delta^2_{31}L}{4E}$$

1.4 Mass effects

When flavour neutrinos pass through matter, they are affected by proton, neutron and electrons present in the matter. Though the first 2 affect all 3 flavours(ν_e, ν_μ, ν_τ) equivalently, so we can ignore that as a phase change, while the electrons only affect electron neutrinos.

Thus, we can imagine Potential Matrix for two-flavoured oscillation to be;

$$\begin{vmatrix} A & 0 \\ 0 & 0 \end{vmatrix}$$

which we will add to the mass Hamiltonian.
Then, the flavour Hamiltonian will be given by;

$$H_f = UH'_m U^\dagger$$

where H'_m is given by adding potential matrix to $H_m = \begin{vmatrix} m_1^2/2E & 0 \\ 0 & m_2^2/2E \end{vmatrix}$ leading to a similar sort of equation. In such a case, the probability of electron neutrino transforming into muon neutrino is given by;

$$P_{e\mu} = Sin^2 2\theta_M Sin^2 \frac{\Delta_M L}{2}$$

where, θ_M is effective mixing angle and Δ_M correspond to eigenvalues of Hamiltonian.

$$\Delta_M = \sqrt{(A - \Delta Cos2\theta) + \Delta^2 Sin^2 2\theta}$$

$$Tan2\theta_M = \frac{\Delta Sin2\theta}{A - \Delta Cos2\theta}$$

On the other hand, for 3-flavours, using $\alpha - s_{13}$ approximations, where we consider these to be small pertubations, we get the following Probabilities for different transformations:[1]

$$P_{ee} = 1 - \alpha^2 \sin^2 2\theta_{12} \frac{\sin^2 A\Delta}{A^2} - 4s_{13}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2},$$

$$P_{e\mu} = \alpha^2 \sin^2 2\theta_{12} c_{23}^2 \frac{\sin^2 A\Delta}{A^2} + 4s_{13}^2 s_{23}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2} + 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta - \delta_{CP}) \frac{\sin A\Delta}{A} \frac{\sin(A-1)\Delta}{A-1},$$

$$P_{e\tau} = \alpha^2 \sin^2 2\theta_{12} s_{23}^2 \frac{\sin^2 A\Delta}{A^2} + 4s_{13}^2 c_{23}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2} - 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta - \delta_{CP}) \frac{\sin A\Delta}{A} \frac{\sin(A-1)\Delta}{A-1}.$$

$$\begin{aligned} P_{\mu\mu} &= 1 - \sin^2 2\theta_{23} \sin^2 \Delta + \alpha\, c_{12}^2 \sin^2 2\theta_{23} \Delta \sin 2\Delta - \alpha^2 \sin^2 2\theta_{12} c_{23}^2 \tfrac{\sin^2 A\Delta}{A^2} - \alpha^2 c_{12}^4 \sin^2 2\theta_{23} \Delta^2 \cos 2\Delta \\ &+ \tfrac{1}{2A} \alpha^2 \sin^2 2\theta_{12} \sin^2 2\theta_{23} \left(\tfrac{\sin\Delta}{A} \cos(A-1)\Delta - \tfrac{\Delta}{2} \sin 2\Delta\right) - 4s_{13}^2 s_{23}^2 \tfrac{\sin^2(A-1)\Delta}{(A-1)^2} - \\ &\tfrac{2}{A-1} s_{13}^2 \sin^2 2\theta_{23} \left(\sin\Delta \cos A\Delta \tfrac{\sin(A-1)\Delta}{A-1} - \tfrac{A}{2} \sin 2\Delta\right) - 2\alpha\, s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos\delta_{CP} \cos\Delta \tfrac{\sin A\Delta}{A} \tfrac{\sin(A-1)\Delta}{A-1} \\ &+ \tfrac{2}{A-1} \alpha\, s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos 2\theta_{23} \cos\delta_{CP} \sin\Delta \left(A \sin\Delta - \tfrac{\sin A\Delta}{A} \cos(A-1)\Delta\right) \end{aligned}$$

$$\begin{aligned} P_{\mu\tau} &= \sin^2 2\theta_{23} \sin^2 \Delta - \alpha\, c_{12}^2 \sin^2 2\theta_{23} \Delta \sin 2\Delta + \alpha^2 c_{12}^4 \sin^2 2\theta_{23} \Delta^2 \cos 2\Delta \\ &- \tfrac{1}{2A} \alpha^2 \sin^2 2\theta_{12} \sin^2 2\theta_{23} \left(\tfrac{\sin\Delta}{A} \cos(A-1)\Delta - \tfrac{\Delta}{2} \sin 2\Delta\right) \\ &+ \tfrac{2}{A-1} s_{13}^2 \sin^2 2\theta_{23} \left(\sin\Delta \cos A\Delta \tfrac{\sin(A-1)\Delta}{A-1} - \tfrac{A}{2} \sin 2\Delta\right) \\ &+ 2\alpha\, s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos\delta_{CP} \cos\Delta \tfrac{\sin A\Delta}{A} \tfrac{\sin(A-1)\Delta}{A-1} \\ &- \tfrac{2}{A-1} \alpha\, s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos 2\theta_{23} \cos\delta_{CP} \sin\Delta \left(A \sin\Delta - \tfrac{\sin A\Delta}{A} \cos(A-1)\Delta\right) \end{aligned}$$

which arise from the following relation between the probabilities;

$$P_{e\tau} = \tilde{P}_{e\mu}, \quad P_{\tau\mu} = \tilde{P}_{\mu\tau}, \quad P_{\tau\tau} = \tilde{P}_{\mu\mu},$$

where;

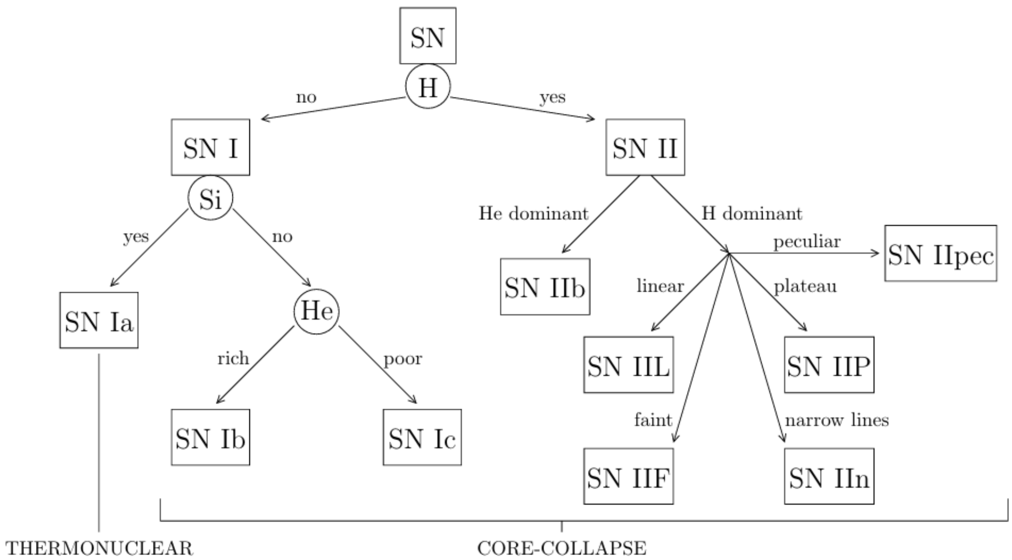
$$\Delta \equiv \frac{\Delta m_{31}^2 L}{4E}, \qquad A \equiv \frac{2EV}{\Delta m_{31}^2} = \frac{VL}{2\Delta}.$$

Chapter 2

Supernovae Neutrino Oscillations

[15]

2.1 Types of Supernovas



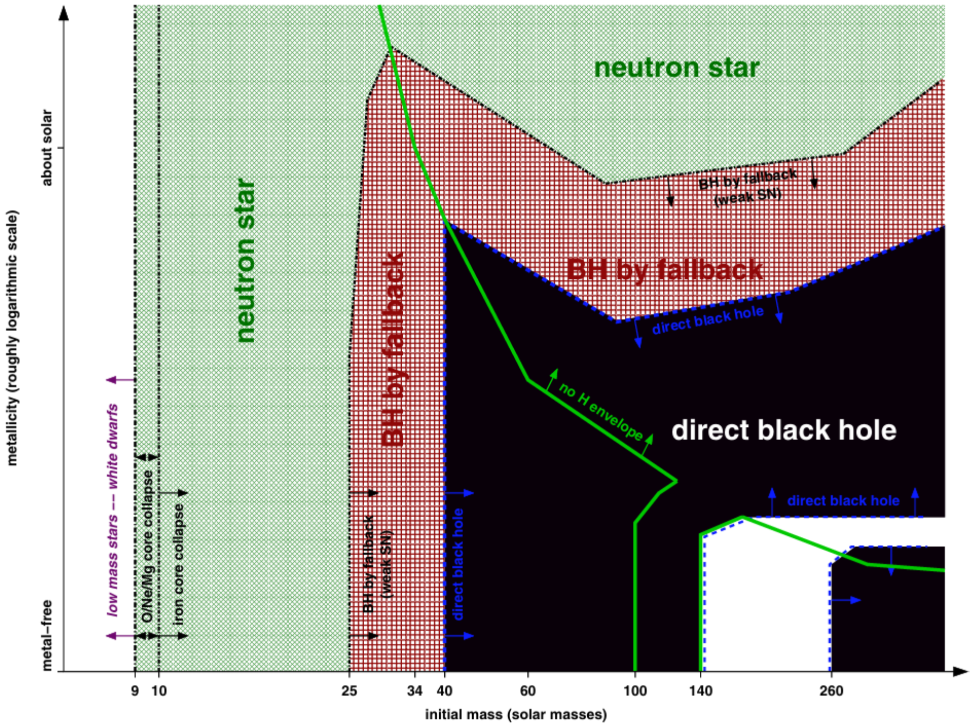
Supernovae are divided on basis of Type-I and II on basis of how much Hydrogem dominates in the ultimate stage of Supernovae.
Type II SNe are thought to be generated by the core collapse of red (or blue as SN1987A) giant stars with a mass between about 8–9 and 40–60 solar masses and thus, we are interested primarily with Type-II and Supernovaes of mass greater than 8-9 Solar masses.
Type II supernovaes are further classified into;

- **IIL**; if the decrease of the luminosity is approximately linear in time; IIP if the time evolution of the luminosity shows a plateau.
- **IIF**; when SNe is faint.
- **IIb**; if helium dominates over hydrogen.
- **IIn** if the spectrum shows narrow line emission.
- **IIpec**; if the SN has peculiar characteristics.

However, we must keep in mind that these classes are not clear-cut and intermediate cases exist.

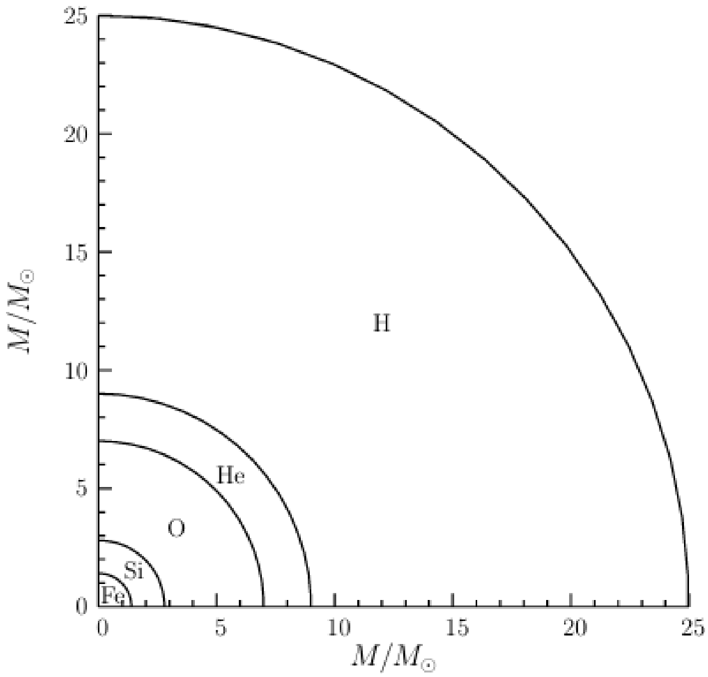
2.2 Core Collapse Mechanism

[7] One can generally predict the end of a star's life based upon Chandrashekar Limit and the war between Gravitational Pressure and Thermonuclear Pressure from which Chandrashekar derives.
Supernovae is basically a loss of balance between the two forces. When the star is active and its fusion process ongoing, the pressure produced by thermonuclear force and gravity of a star, due to its own mass balance each other. But as time goes on, the H is exhuasted first, followed by He, then O, Si and finally Iron.
But, Iron is a stable element in either fission and fusion i.e. it takes much more energy to drive the process than the energy produced, and thus thermonuclear reaction comes to a halt. This leads to the star imploding upon itself, as the Energy is inversely proportional to radius and the numerator is negative in this case. So to stabilize the stars, r drops rapidly. This event might produce Black Hole for which the mass was calculated by Dr. Chandrashekar and was named after him as Chandrashekar Limit[1], about 1.4 Solar masses. Other than S. Chandrashekar, physicist Lev Landau also calculated it using atomic structure and formulating parallels between the two systems



As one may notice the image above represents the fate of stars as per their solar masses. We are majorly interested in the green area. Just before the thermo-nuclear fuel runs out, we get an onion structure of the star according to different elements but as it runs out the stars implodes upon itself, reducing to a fraction of its radius. During this sudden shock some elements of the star is left out which are called "Envelope" as it envelopes the proto-neutron star. In the Red region, these fallback into the dense proto-neutron star[formulated majorly by neutron and thus extremely dense] transforming into a black hole.

[1] S. Chandrasekhar, An Introduction to the Study of Stellar Structure, University of Chicago Press, 1938



During this core-collapse, a few neutrinos amongst about 10^{58} neutrinos escape and we notice the first bump in neutrino flux graph, warning us of SNe. But after that, the rest of the neutrinos get captured by the dense envelope and we do not see neutrino flux for some time.
But less than one percent neutrinos have escaped before envelope closes them off. The shock which had become still takes energy from a few of these neutrinos(about 5-8 per cent) and the shock revives and a huge flux of neutrinos is observed within a span of 10sec.
The Energy liberated during SNe is released primarily in form of neutrinos(99 percent), ElectroMagnetic Radiation and other matter particles. While light is trapped inside the Core and releases slowly over the years, the neutrinos because of their weakly interacting nature act as harbingers of doom, bringing with themselves the news of SNe as well as immense amount of data in extremely short time.

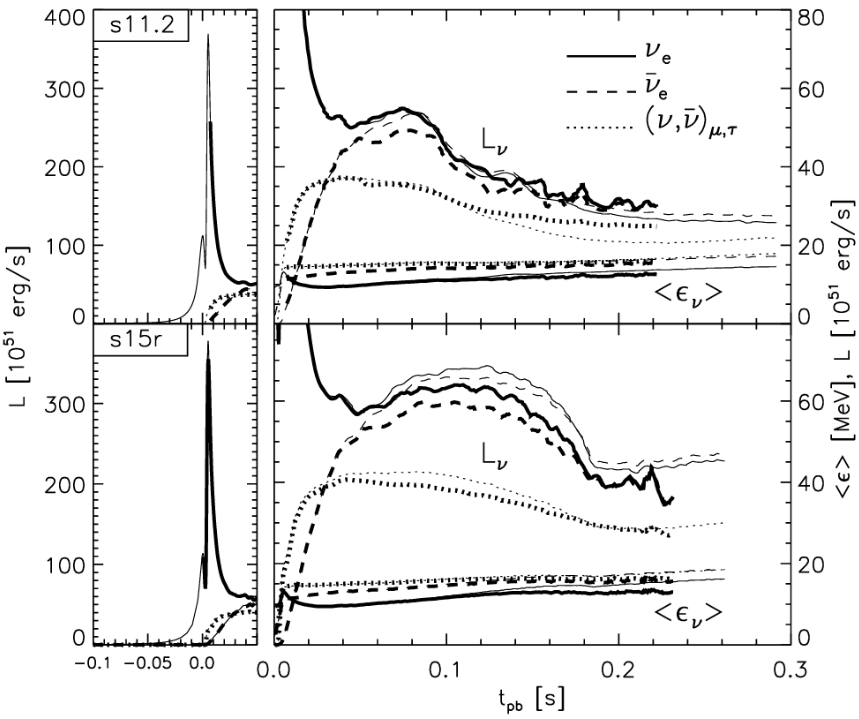
2.3 Supernova Neutrinos

[6] Throughout the Supernova, all 3 flavour neutrinos and antineutrinos are created. Because, the proto neutron star is dense in nucleons, we can club neutrinos based upon their interactions with matter i.e. $\nu_e, \overline{\nu_e} and \nu_x$ which contains both neutrinos and antineutrinos of tau and muon neutrinos. This is because electron neutrino interact differently with matter than electron neutrino and both of them interact differently than the other four, which interact similarly.
Hence, we can assume 3 neutrinospheres according to mean free length due to interactions;

- ν_e (interacts with p,n,e)
- $\overline{\nu_e}$ (interacts with p,n in a similar manner but interacts differently with e)
- ν_x (interacts only with p,n in a similar manner)

In the compact remnant of SNe[proto-neutron star in most cases] hot neutrinos of all types are produced. Remnant and Surroundings envelope is optically thick, not allowing light and neutrinos to release. They carry about $3x10^{53}$erg of Energy which is about 99 per cent of binding gravitational energy.

About10^{58} neutrons of average energy, $\langle E\rangle = 10MeV$ is released, carrying away most of Energy and lepton number with order of magnitude is greater than lepton number of collapsed core.



During the process of core collapse, the mechanism of neutrino production is by ;

- electron capture of nuclei

$$e^- + N(Z,A) \rightarrow N(Z-1,A) + \nu_e$$

- free proton

$$e^- + p \rightarrow n + \nu_e$$

which are favored by high electron Energy of formation and thus reduce the number of electrons. At the very start when density of Iron core is not that high, ν_e is produced by electron capture which takes Energy away and lowers electron pressure, reducing Chandrashaker mass[2] untill it is less than mass of core.
The electron neutrino produced in electron capture has average energy of about 12 MeV and luminousity of about 10^{53} erg/s, bout out of that only 1 per cent are able to escapes before they are enveloped as 'Core Bounce' lasts only about **10 ms**.
After that and in proto-neutron star state, all varieties of neutrinos are produced in the core of SNe, by following 4 processes;

- $e^{\pm} + N \rightarrow e^{\pm} + N + \nu + \overline{\nu}$

- $N + N \rightarrow N + N + \nu + \overline{\nu}$

- Plasmon Decay process

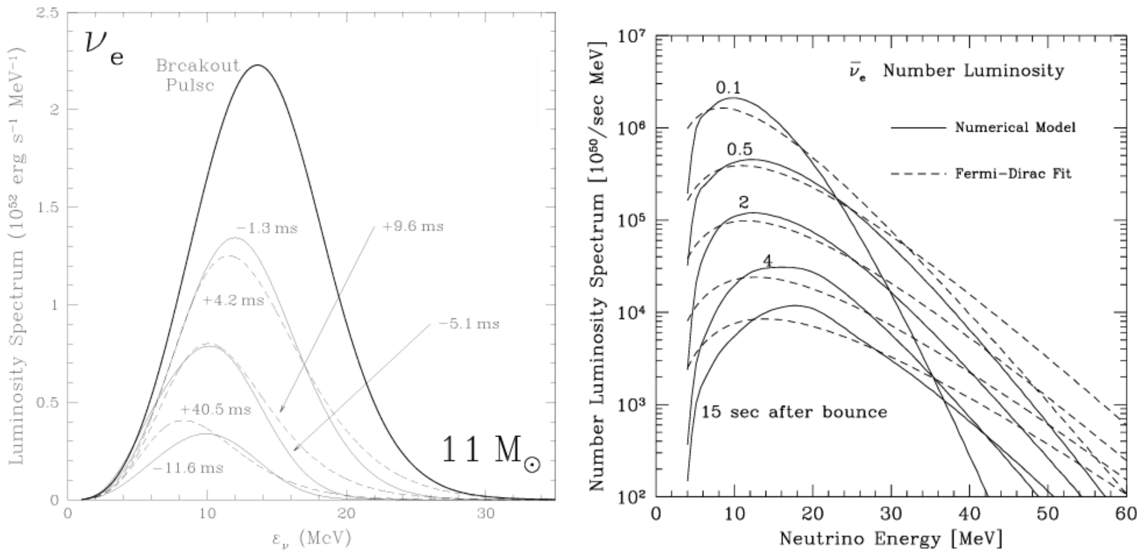
$$\gamma \rightarrow \nu + \overline{\nu}$$

- Photo-annihilation Process

$$\gamma + e^{\pm} \rightarrow e^{\pm} + \nu + \overline{\nu}$$

[2] $M_c = 2.10^{57} Y_e^3$,where Y_e is electron fraction

while ν_e is also produced by electron capture, while $\overline{\nu_e}$ by $e^+ + n \rightarrow p + \overline{\nu_e}$.
On one hand p,n,e doesn't allow creation of μ and τ, hence we get 3 different neutrinospheres(r about 50-100Km) for every energy, as mentioned earlier with electron neutrinosphere having the largest radius for same energy, and x having the smallest.



Rough estimates of average energy is 10,15 and 20MeV respectively for $\nu_e, \overline{\nu_e}$ and ν_x respectively having a graph similar to Fermi-Dirac Statistics.

2.4 SN1987A

SN-SuperNovae
1987-Year
A-First SNe in the year

is the only supernova in recent memory which has been documented in details far-exceeding the visual details. 3 observatories; **Kamiokande-II[5], IMB[2] and Baksan[12]** observed an unusual number of events about the same time within a span of 10 seconds.
The observed events were in agreement with proposed model of Core-collapse and Core cooling by neutrinos in every respect. Thus, from the data collected by these three observatories, physicists were able to bound;

- **Electron Neutrino Mass**

$$m_{\nu_e} < 5.7eV$$

- **Mass State neutrino Mass**

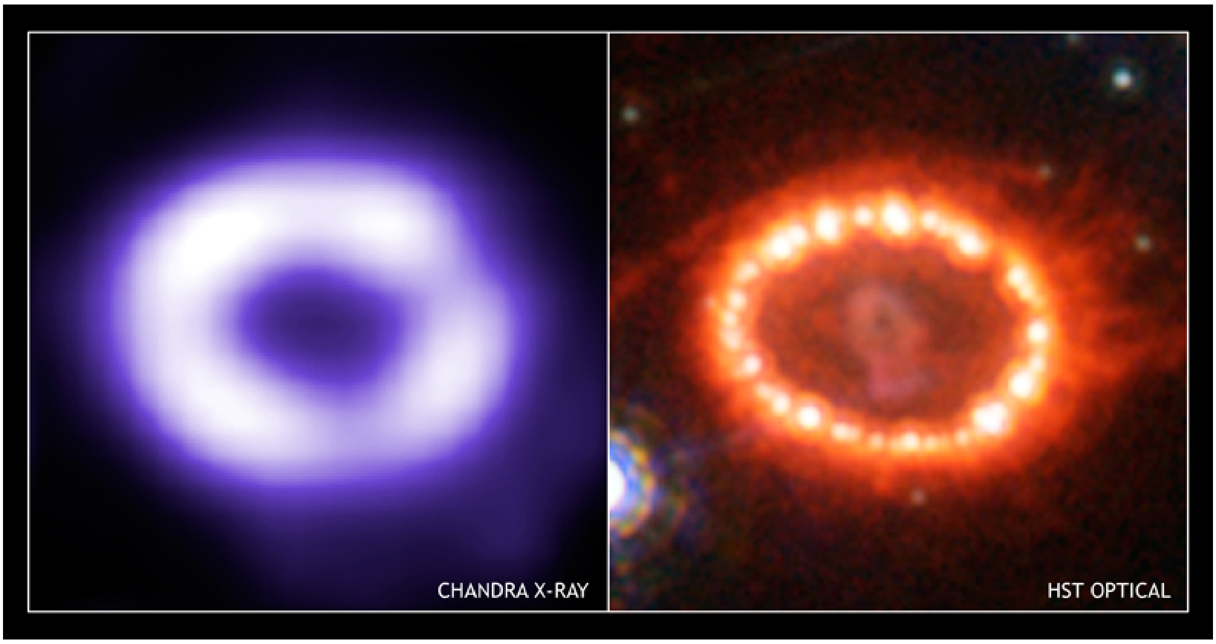
$$m_k < 5.7eV \quad \text{where k=1,2,3}$$

- **Heavy neutrino mass**

$$m_h \leq 30eV or m_h \geq 1.1KeV$$

- **Large $\overline{\nu_x} \rightleftharpoons \overline{\nu_e}$ transitions disfavoured**

- **Charge Radius for Right Handed Neutrinos**[3]
  because of abscence of cooling constraint in our theory, we get $\langle r^2 \rangle_R \leq 2.10^{-33} cm^2$



2.5 Mathematical Tool: Lindblad Equation(OQS)

[11] For an open quantum system, we can describe the system using density matrix, ρ which is described such that it's diagonal elements refer to probabilities of respective states[Tr(ρ)]=1 whereas, the off diagonal elements are a representative of decoherence.
For a system interacting with the environment, we can describe its interaction in terms of ρ and H using Linbland Master Equation;

$$\frac{d}{dt}\rho = i[\rho, H] + D$$

,where D is a trace-one, linear operator acting on H and H is the Hamiltonian.
A few of the notable properties of density matrix (ρ) is;

- $Tr[\rho] = TotalProbability = 1$
- $Tr[\rho^2] \leq 1$
- if Tr[ρ^2]=1, the system is a pure system.
- Off diagonal elements represents interchanging of one state into another(coherence).
- $\langle A \rangle = Tr[\rho A]$

This shall be extremely useful in analyzing Probabilities of Supernovae Neutrinos which can be accounted easier using Denisty Matrix[OQS] approach.[16][3]
As more and more hamiltonian terms get introduced and system gets more and more complex, we are better off with Linbland equation. In general, we can write Transformation probability of some flavour α to β as;

$$P_{\alpha\beta} = \sum_{i,j,k=1}^{3} P_{\alpha i}^{m(\mathrm{SN})} P_{ij}^{m(\mathrm{SN})} P_{jk}^{m} P_{k\beta}^{m(\mathrm{Earth})} \tag{2.1}$$

[3] Charge radius is a measure of how far the electric charge of a particle is spread out from its center.

$$\bar{P}_{\alpha\beta} = \sum_{i,j,k=1}^{3} \bar{P}_{\alpha i}^{m(\mathrm{SN})} \bar{P}_{ij}^{m(\mathrm{SN})} \bar{P}_{jk}^{m} \bar{P}_{k\beta}^{m(\mathrm{Earth})} \tag{2.2}$$

where i,j are mass states(1,2,3) and α, β are the flavour states(e, μ, τ). 'm' refers to matter, here. Thus, we end up probabilities for both neutrino and it's antineutrino traveling through space.
Our next step must be to calculate these Probabilties and plot them to understand the phenomenonology of Supernova Neutrinos better. For that, one can study

- **Paper on SN neutrino Probability by Dr. Amol DIghe**[3]
- **Understand Effects of Quantum Decoherence in Supernovae Neutrinos**[16]

of which the latter uses OQS and Master equation approach whereas the former tries to approach using flux of respective neutrinos which are, both intuitively and physically directly correlated to respective Probabilities.

Chapter 3

Neutrino Oscillations and Resonance

3.1 Resonance

[10]

In the 80s, Mikheyev and Smirnov proposed a new approach targeted to solar neutrino flux [13, 14] where even the small mixing angles can lead to a sizable reduction of solar neutrino flux through resonant neutrino oscillations inside the sun, especially driving from charged current contributions for various neutrino flavours propogating through matter.

We know evolution for 2-flavour Neutrino oscillation in matter can be written as;

$$i\frac{d}{dt}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \frac{1}{2E}\begin{pmatrix} m_1^2 + \Delta m^2 \sin^2\theta + D & \Delta m^2 \sin\theta\cos\theta \\ \Delta m^2 \sin\theta\cos\theta & m_1^2 + \Delta m^2 \cos^2\theta \end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}.$$

which when compared to 2-flavour neutrino oscillation in matter, we get mixing angle as a function of D.

$$\sin^2 2\theta_m = \frac{(\Delta m^2)^2 \sin^2 2\theta}{(D - \Delta m^2 \cos 2\theta)^2 + (\Delta m^2 \sin 2\theta)^2}.$$

where θ is solar mixing angle and Δm^2 is mass difference between the mass states.

Hence, even for small vacuum mixing angle $Sin^2\theta_m$ shows resonance as function of D. As $Sin^2\theta_m$rises from its vacuum value to 1 when D increases through resonance. Hence at high density(far beyond resonance density $D_r = \Delta m^2 Cos2\theta$) electron neutrino corresponds to ν_2 as θ_m becomes $\pi/2$ and muon into ν_1 completely. Thus the MSW effect results from the resonance behaviour of mixing angle.[17]

A similar thing happens in 3 flavoured case which shall be discussed in the next section. The results are plotted on the next page [1]

Also for 3-flavour neutrino oscillation, we may write the Schrodinger like ultrarelativistic equation as:

$$i\frac{d}{dt}\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} = \frac{1}{2E}\begin{pmatrix} m_1^2 + Dc_{12}^2 c_{13}^2 & Dc_{12}s_{12}c_{13}^2 & Dc_{12}c_{13}s_{13}e^{-i\delta} \\ Dc_{12}s_{12}c_{13}^2 & m_2^2 + Ds_{12}^2 c_{13}^2 & Ds_{12}c_{13}s_{13}e^{-i\delta} \\ Dc_{12}c_{13}s_{13}e^{i\delta} & Ds_{12}c_{13}s_{13}e^{i\delta} & m_3^2 + Ds_{13}^2 \end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix}.$$

[1] Taken from Zaugler's 1988 paper[17]

You may see the result below

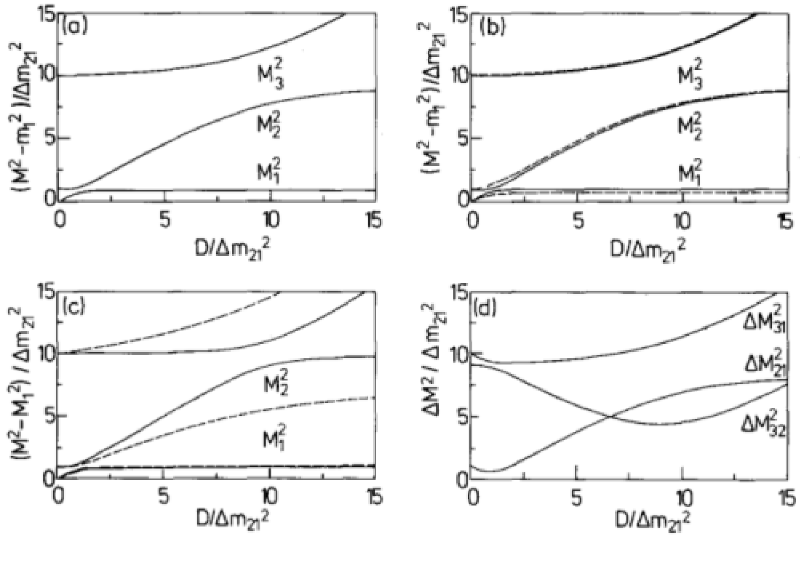


Fig. 1 a–d. Mass eigenvalues in matter for three neutrino generations with clearly separated neutrino masses ($\Delta m^2_{31}/\Delta m^2_{21} = 10.0$) and various vacuum mixing angles: **a** $s^2_{12} = 0.05$, $s^2_{13} = 0.05$, **b** $s^2_{13} = 0.05$; $s^2_{12} = 0.01$ (solid line) and $s^2_{12} = 0.2$ (dashed line), **c** $s^2_{12} = 0.05$; $s^2_{13} = 0.01$ (solid line) and $s^2_{13} = 0.2$ (dashed line). In **d** the mass differences for the masses in matter are shown for $s^2_{12} = 0.05$ and $s^2_{13} = 0.05$

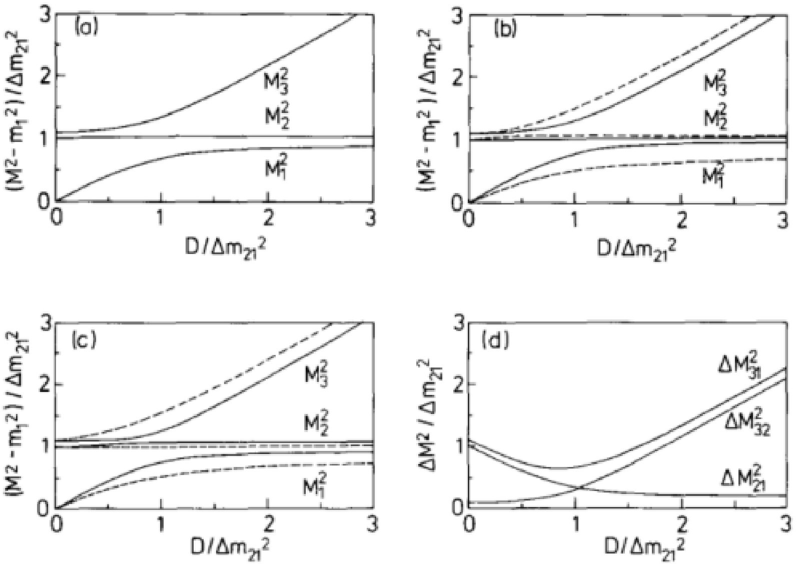


Fig. 2a–d. Mass eigenvalues in matter for three neutrino generations with almost degenerate neutrino masses ($\Delta m^2_{31}/\Delta m^2_{21} = 1.1$) and various vacuum mixing angles: **a** $s^2_{12} = 0.05$, $s^2_{13} = 0.05$, **b** $s^2_{13} = 0.05$; $s^2_{12} = 0.01$ (solid line) and $s^2_{12} = 0.2$ (dashed line), **c** $s^2_{12} = 0.05$; $s^2_{13} = 0.01$ (solid line) and $s^2_{13} = 0.2$ (dashed line). In **d** the mass differences for the masses in matter are shown for $s^2_{12} = 0.05$ and $s^2_{13} = 0.05$

3.2 Neutrino Oscillations in Neutrino Gases

[8] Self-maintained coherent neutrino oscillations are usually occur when the gas density exceeds a critical value determined by the neutrino masses and the mean neutrino energy in the gas. For simplicity let's just consider 2 solar neutrinos, considering neutrinos are neither created nor destroyed in the process and just oscillate. Then the relevant parameters would be the mixing angle and mass square difference, $\Delta = (m_2^2 - m_1^2)c^2$.

We can represent the mass states as;

$$\nu_1 = \nu_e \cos\theta - \nu_\mu \sin\theta, \qquad \nu_2 = \nu_e \sin\theta + \nu_\mu \cos\theta,$$

Vacuum-oscillation Time period shall be given by;[4]

$$T = \frac{4\pi E\hbar}{\Delta}$$

Vacuum behavior dominates provided the neutrino gas is sufficiently dilute. The dominance is controlled by[2]

$$\kappa = \frac{\Delta}{2\sqrt{2}G_F E n_\nu},$$

where n_ν is neutrino density. We may write this parameter as ratio T_ν/T when;

$$T_\nu = \frac{2\pi\hbar}{\sqrt{2}G_F n_\nu},$$

Hence, when the neutrino density is low, κ is large. Here, in vacuum the behaviour is similar to non-interacting gas as prob of creation and destruction is quite less.
when the neutrino density is large so that κ 1, neutrino interactions are important. Many neutrino-neutrino interactions occur during a vacuum oscillation period.
Significant numbers of neutrinos oscillate in unison. We refer to this behavior of neutrinos as self-maintained coherence. A system consisting initially of electron neutrinos does not decohere. Instead, oscillatory behavior is observed, even at late times.

3.3 Supernova Neutrino Oscillation

[9] During first few milliseconds of the neutrino burst from a supernova, the signal is expected to be dominated by the ν_e**??**, which are produced by the electron capture on protons and nuclei while the shock wave passes through the neutrinosphere [32]. Since the original flux is known, the observed fluxes will give a measurement of conversion of ν_e into the other neutrino species.
Moreover in the neutrino flux graph, we hope to see a pinched spectrum because of temperature in SNe decreasing with increasing radius and density decreasing faster than 1/r. One way to parametrize the pinched neutrino spectra is to introduce an effective temperature T_i and an effective degeneracy parameter η_i such that fermi dirac statistics follow the formula;

$$F_i^0(E) \propto \frac{E^2}{\exp\left(E/T_i - \eta_i\right) + 1}$$

where i represents the flavour. The value of η_i is same for all the x neutrinos and neutrinos and different from e due to similar interactions. Typically,

$$T_e \approx 3-4 \text{ MeV}, \qquad T_{\bar{e}} \approx 5-6 \text{ MeV}, \qquad T_x \approx 7-9 \text{ MeV}$$

and

$$\eta_e \approx 3-5, \qquad \eta_{\bar{e}} \approx 2.0-2.5, \qquad \eta_x \approx 0-2$$

[2] Refer to 2 flavour neutrino oscillation in Matter

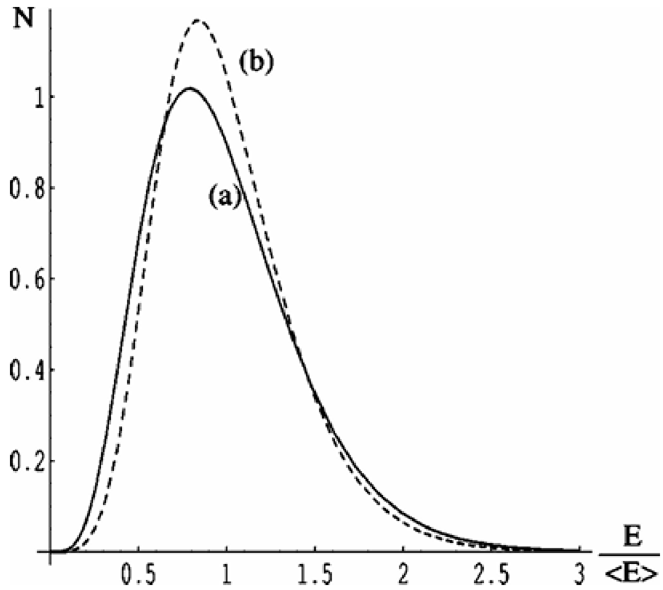


FIG. 1. The number of $\nu - N$ charged current events for (a) a thermal spectrum $(T=3, \eta=0)$, (b) a "pinched" spectrum $(T=3, \eta=3)$. T and η are the parameters as in Eq. (7). Both the energy resolution function and the lower energy threshold effects have been taken into account. The spectra are normalized to have equal areas.

In supernova, the transitions occur mainly in the resonance layers which can be characterized in 2; one with higher densities(corresponding to Δm^2_{atm})

$$\rho_H = 10^3 - 10^4 g/cc$$

and one with lower densities(characterized by Δm^2_{Solar}

$$\rho_L = \begin{cases} 5-15 \text{ g/cc} & \text{for Small Solar Mixing Angle,} \\ 10-30 \text{ g/cc} & \text{for Large Solar Mixing Angle,} \\ < 10^{-4} \text{ g/cc} & \text{for Vacuum Oscillation.} \end{cases}$$

Hence, we see in H-resonance region

$$\frac{U^m_{e2}}{U_{e2}} \sim \frac{\rho_L}{\rho_H} \lesssim 10^{-2}.$$

i.e. there is a suppression.

Whereas in the L-resonance region, mixing associated with atmospheric coincides with vacuum mixing i.e. $U^m_{e3} = U_{e3}$ Hence the mixing associated with atmospheric is almost constant and ν_3 will not participate in oscillation.

Given all that, if we calculate Conversion Probabilities and Neutrino Fluxes.

The neutrino flux arise from the central part of the star where $\rho >> \rho_H, \rho_L$ i.e. all mixings are highly supressed. Hence,

$$\nu_{3m} = \nu_e, \qquad \nu_{2m} = \nu_{\tau'}, \qquad \nu_{1m} = \nu_{\mu'}$$

and

$$F^0_{1m} = F^0_x, \qquad F^0_{2m} = F^0_x, \qquad F^0_{3m} = F^0_e$$

If we calculate flux of first eigenstate ν_1 at surface of the star. The probability of original state ν_e arrives at surface as it will be $P_H P_L$ since it has to flip to other matter eigenstate at both resonances, i.e. the flux would be $F^0_e P_H P_L$, then the flux from muon neutrino would be $(1-P_L)F^0_x$ and from tau would be $P_L(1-P_H)F^0_x$. Hence,

$$F_1 = P_H P_L F^0_e + (1 - P_H P_L) F^0_x.$$

and similarly;

$$F_2 = (P_H - P_H P_L) F^0_e + (1 - P_H + P_H P_L) F^0_x$$

$$F_3 = (1 - P_H) F^0_e + P_H F^0_x.$$

generally, we can write them as

$$F_i = a_i F^0_e + (1 - a_i) F^0_x,$$

where

$$a_1 = P_H P_L, \qquad a_2 = P_H(1 - P_L), \qquad a_3 = 1 - P_H.$$

We know that

$$F_z = \sum_i |U_{zi}|^2 F_i$$

where z is some flavour state and i represents mass states.
For electron neutrino, we can say,

$$F_e = F^0_e \left[\sum_i |U_{ei}|^2 a_i + F^0_\chi \left(1 - \sum_i |U_{ei}|^2 a_i \right) \right]$$

Hence the electron reaching earth can be written as

$$F_e = p F^0_e + (1 - p) F^0_x$$

with p as

$$\begin{aligned} p &= \sum_i |U_{ei}|^2 a_i \\ &= |U_{e1}|^2 P_H P_L + |U_{e2}|^2 (P_H - P_H P_L) + |U_{e3}|^2 (1 - P_H). \end{aligned} \tag{3.1}$$

and by conservation of flux

$$F_\mu + F_\tau = (1 - p)\, F^0_e + (1 + p)\, F^0_\chi. \tag{3.2}$$

which gives us the final values of their fluxes, which will help us compute and compare the fluxes obtained in SNe neutrino detectors.[3]

[3] **Note:** we have not considered matter effects due to earth but they are easily computable since we know the fluxes at surface of the earth and are well aware of matter effects.

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